7 QUANTUM NOISE + OPTICAL LAYOUT
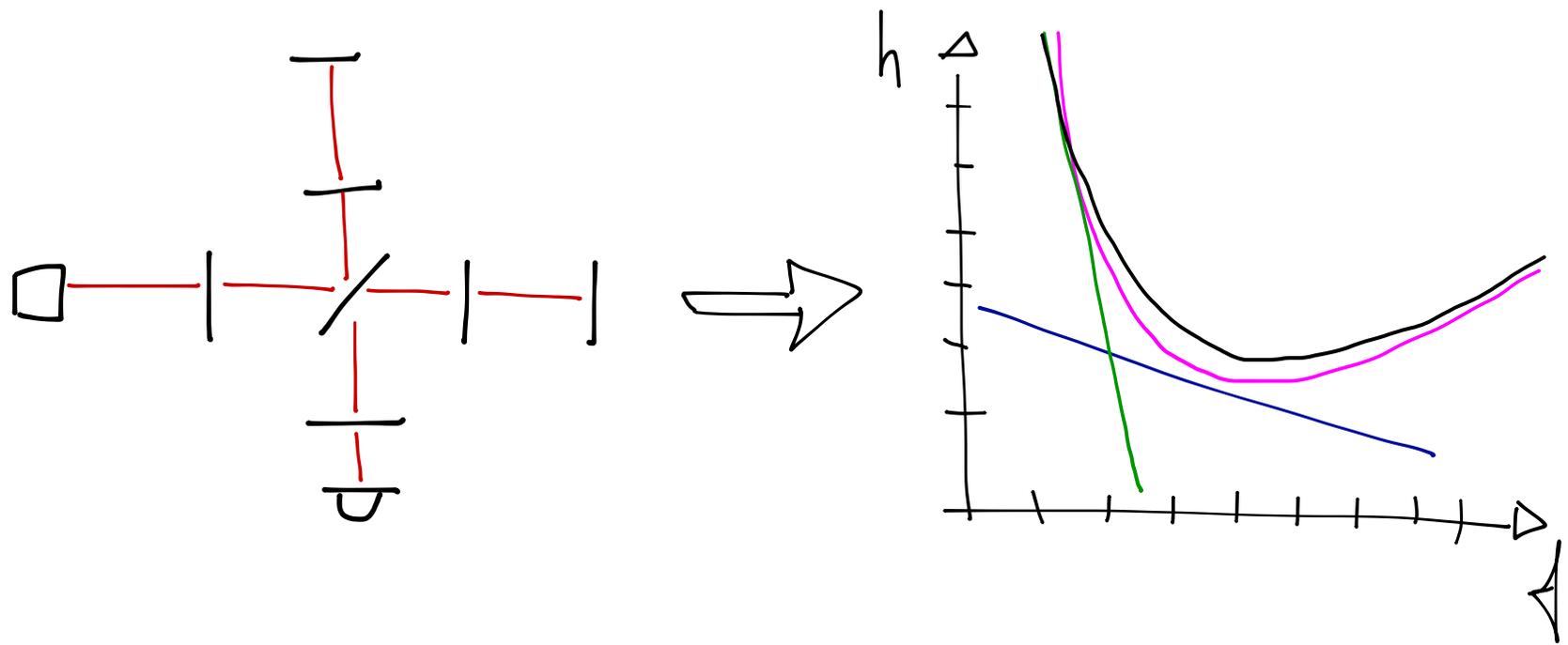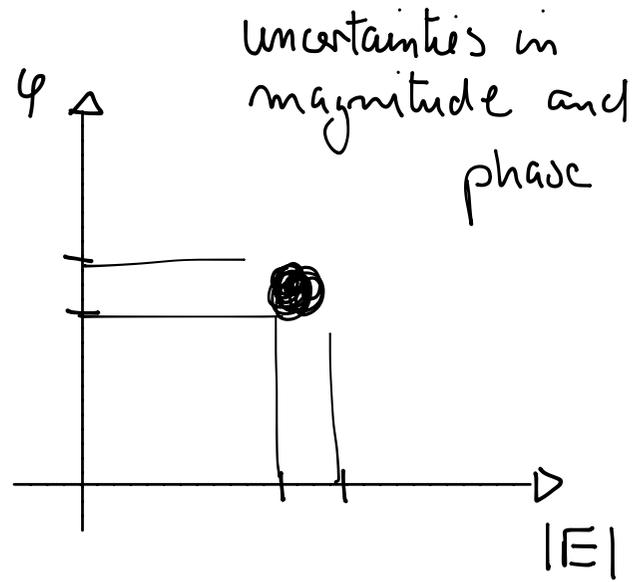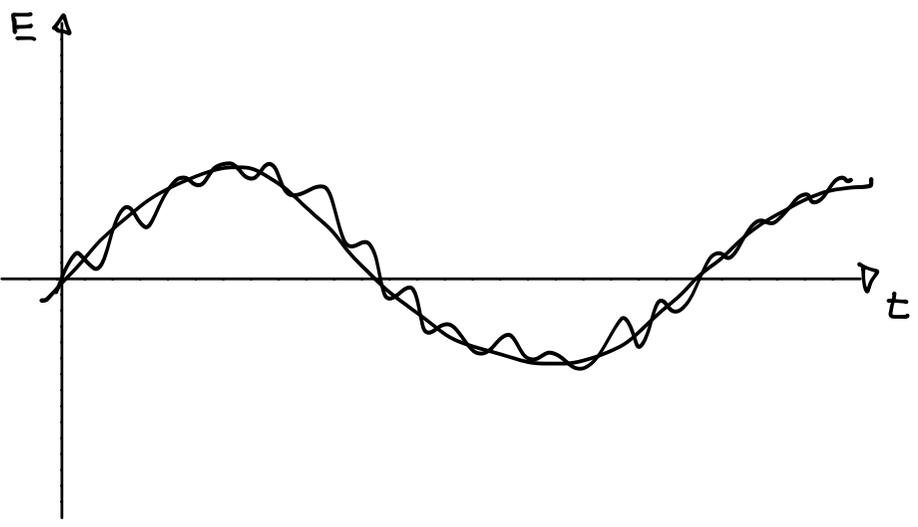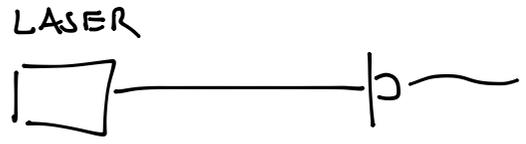
This Session:

- The optical layout of Advanced LIGO
- The reason for the addition of extra components
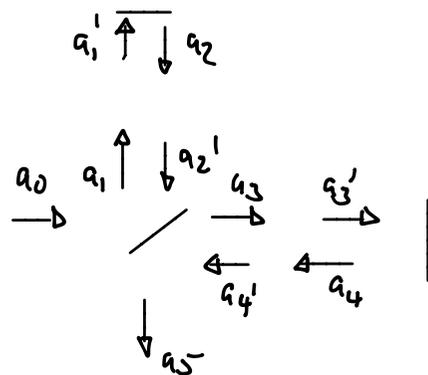 (quantum noise)

Quantum noise

A perfect laser

LASER

E

t

uncertainties in magnitude and phase

φ

$|E|$

PSD of quantum noise $S_q = 2 P_0 \hbar \omega_0$

GW signal strength in Michelson output

$$a_1' = \sqrt{\tfrac{1}{2}}\, e^{-ikL_1}\, a_0$$

$$a_2 = \boxed{a_1'} \cdot (1 + im\cos(\omega_m t))$$

$$a_2' = \sqrt{\tfrac{1}{2}}\, a_0\, e^{-i2kL_1}\, (1 + im\cos(\omega_m t))$$

$$a_4' = i\sqrt{\tfrac{1}{2}}\, a_0\, e^{-i2kL_2}\, (1 - im\cos(\omega_m t))$$

GW is differential like ΔL

$$a_5 = \sqrt{\tfrac{1}{2}}\left(i a_2' + a_4'\right) = \tfrac{1}{2} i a_0\, e^{-i2k\bar{L}}\left(e^{-i2k\Delta L}(1 + im\cos()) + e^{i2k\Delta L}(1 - im\cos())\right)$$

$$= i a_0\, e^{-i2k\bar{L}}\left(\cos(2k\Delta L) + \tfrac{i}{2}m\cos()\underbrace{\left(e^{-i2k\Delta L} - e^{i2k\Delta L}\right)}_{= -2i\sin(2k\Delta L)}\right)$$

$$= i a_0\, e^{-i2k\bar{L}}\cos(2k\Delta L) + a_0 m\, e^{-i2k\bar{L}}\sin(2k\Delta L)\cos(\omega_m t)$$

Usually we now do P_5 to compute the signal, but for quantum noise we better look a bit closer.

Signal and local oscillator

at dark fringe, $\cos(2k\Delta L) = 0$

$\Rightarrow a_S = a_0 \, m \, e^{-i2kL} \cos(\omega_m t)$

$P_S = P_0 \, m^2 \cos^2(\omega_m t) \Rightarrow$ No signal at ω_m

We add a 'local oscillator' a_{LO}:

$a_S = a_{LO} + a_0 \, m \, e^{-i2kL} \cos(\omega_m t)$

$P_S = P_{LO} + \boxed{Re\{a_{LO} \, a_0^* \, e^{-i2kL}\} \cdot m \cdot \cos(\omega_m t)}$
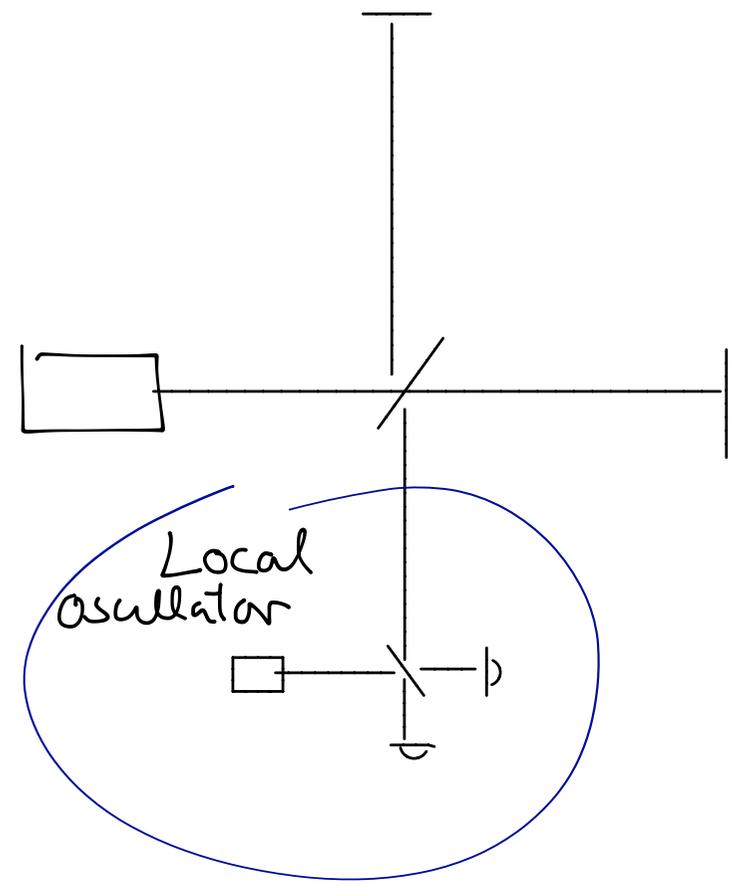
$+ P_0 \, m^2 \cos^2(\omega_m t)$

$\sim a_{LO} \, a_0 \, m \cdot \cos(\omega_m t)$

Quantum noise: extra modulation at the output

$a_S = a_{LO} + a_0 \, m \, e^{-i2kL} \cos(\omega_m t) + n \cos(\omega_m t)$

$\Rightarrow P_S(\omega_m) \sim a_{LO} a_0 \, m \cos(\omega_m t) + a_{LO} \, n \cos(\omega_m t)$

Local oscillator

$\dfrac{}{}4$

$$P_S(\omega_m) \sim a_{LO} a_0 \, m \cos(\omega_m t) + a_{LO} n \cos(\omega_m t)$$

GW amplitude   fixed by Nature

$\sqrt{\text{Laser power}}$!

To be precise: $\sqrt{P_A}$ with P_A the power at the moment of modulation
i.e. the power in the arm!

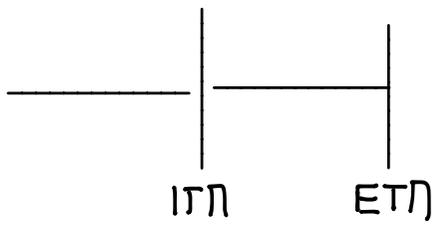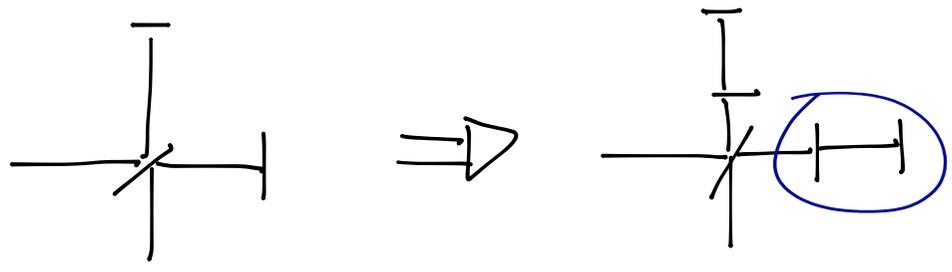Conclusion: the win over quantum noise we have to improve power in the arms.
Signal to quantum noise increases by $\sqrt{P}$

Note this covers only 'shot noise' and not 'radiation pressure noise' or any complicated
quantum noise reduction scheme, but is sufficient to explain the Advanced LIGO optical layout.

Increasing the power

0. bigger laser ! ☺

1. arm cavities

$R_{ETM} \sim 1$

$R_{ITM} = 0.986 \quad , \quad T_{ITM} = 0.014$

Power increase $\approx \dfrac{4}{T_{ITM}} \approx 300$

ITM : Input test mass
ETM : End test mass

Why not more? I.e. why not $T = 0.00001$?

⇒ Techical difficulties in making and opesating such a high finesse cavity.
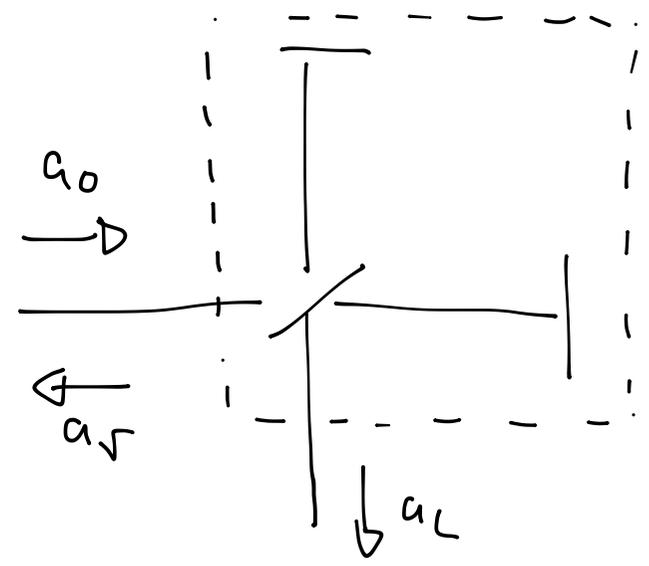
More power through recycling!

$$a_t = i a_0 e^{-i2k\bar{L}} \cos(2k\Delta L)$$

$$a_r = -a_0 e^{-ik2\bar{L}} \sin(2k\Delta L)$$

Define $r_{MI} = \dfrac{a_r}{a_0}$

$\Rightarrow R_{MI} = \sin^2(2k\Delta L)$. At dark fringe $2k\Delta L = \dfrac{\pi}{2} \Rightarrow \sin^2( ) = 1$

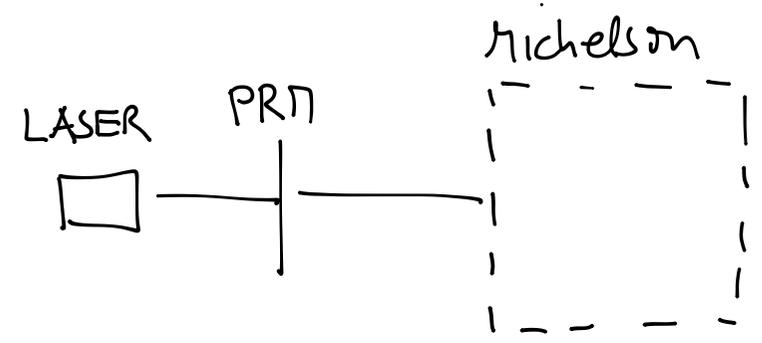$\Rightarrow \underline{R_{MI} = 1}$ all power is reflected back to the laser!
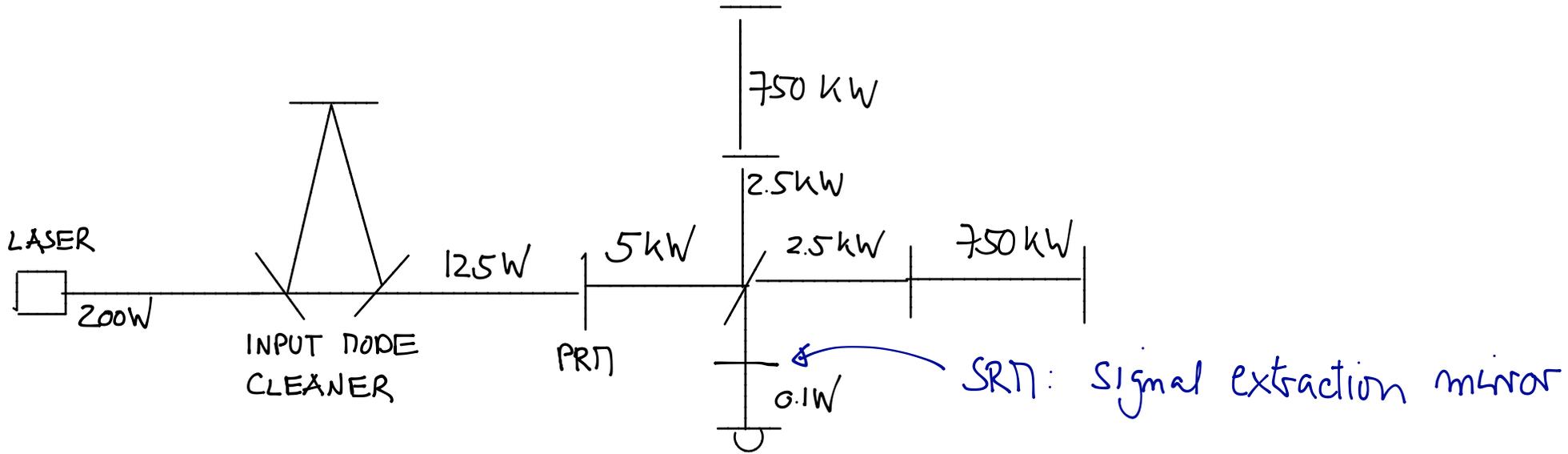
Lets make another cavity!

PRM: Power Recycling Mirror

$T_{PRM} \sim 0.1$,

Power increase $\sim \dfrac{4}{1} = 40$

a_0

a_r

a_L

Michelson

LASER   PRM

$\dfrac{7}{}$

Light powers in LIGO

LASER

200W
INPUT MODE
CLEANER

125W

PRM

5kW

2.5kW

750 KW

2.5 kW

750 kW

0.1W

SRM: Signal extraction mirror

Signal to noise improvement is $\sqrt{\dfrac{750\,k}{125}} \approx 80$

Summary:

- Why high power? Quantum noise!

- Arm cavity power increase ~ 300

- Power recycling: increase ~ 40, requires dark fringe

Signal-to-quantum noise $\sim \sqrt{P}$